2. Trigonometric Functions

1. If A(x, y) is any point on the terminal arm OQ such that OA = $r = \sqrt{x^2 + y^2}$ and ∠POQ = q then:

$\sin q = \frac{y}{r}$

$\cos q = \frac{x}{r}$

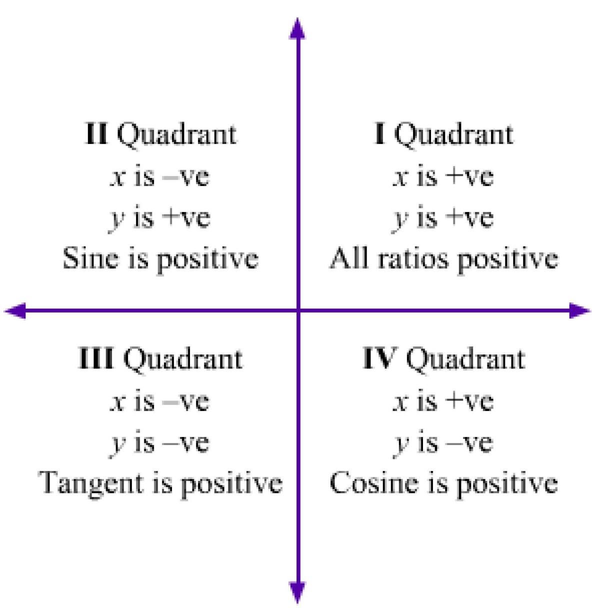
$\tan q = \frac{y}{x}$, where $x \neq 0$

$\text{cosec}\ q = \frac{r}{y}$, where $y \neq 0$

$\sec q = \frac{r}{x}$, where $x \neq 0$

$\cot q = \frac{x}{y}$, where $y \neq 0$

2. The signs of various trigonometric ratios in different quadrants are as follows:



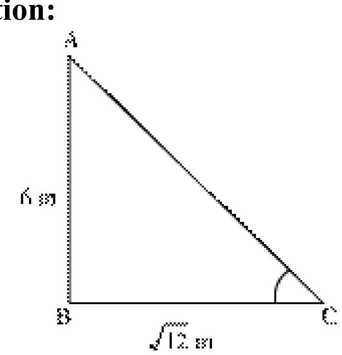
- **Trigonometric Ratios of some specific angles**

q	0	30°	45°	60°	90°
sin$q$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos$q$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan$q$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec$q$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec$q$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot$q$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example 1:

ΔABC is right-angled at B and AB = 6 m, $\mathbf{BC} = \sqrt{12}$ m. Find the measure of ∠A and ∠C.

Solution:



$$AB = 6 \text{ m},$$

$$BC = \sqrt{12} \text{ m} = 2\sqrt{3} \text{ m}$$

$$\tan C = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \tan C = \tan 60° \qquad \left[\because \tan 60° = \sqrt{3}\right]$$

$$\Rightarrow \angle C = 60°$$

$$\therefore \angle A = 180° - (90 + 60) = 30°$$

Example 2:

Evaluate the expression

$$4\left(\cos^3 60° - \sin^3 30°\right) + 3\left(\sin 30° - \cos 60°\right)$$

Solution:

$$4\left(\cos^3 60° - \sin^3 30°\right) + 3\left(\sin 30° - \cos 60°\right)$$

$$= 4\left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3\right] + 3\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= 4 \times 0 + 3 \times 0 = 0 + 0 = 0$$

- **Trigonometric Identities**
  1. $\cos^2 A + \sin^2 A = 1$
  2. $1 + \tan^2 A = \sec^2 A$
  3. $1 + \cot^2 A = \operatorname{cosec}^2 A$

Example:
If $\cos\theta = \frac{5}{7}$, find the value of cot θ + cosec θ

Solution:
We have, $\cos\theta = \frac{5}{7}$

Now, $\sin^2\theta + \cos^2\theta = 1$

$$\therefore \sin\theta = \sqrt{1 - \cos^2\theta}$$

$$= \sqrt{1 - \left(\frac{5}{7}\right)^2}$$

$$= \sqrt{\frac{49 - 25}{49}} = \frac{2\sqrt{6}}{7}$$

$$\therefore \operatorname{cosec}\theta = \frac{7}{2\sqrt{6}}$$

$$\text{Also, } \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\frac{5}{7}}{\frac{2\sqrt{6}}{7}} = \frac{5}{2\sqrt{6}}$$

$$\therefore \cot\theta + \operatorname{cosec}\theta = \frac{5}{2\sqrt{6}} + \frac{7}{2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \sqrt{6}$$

1. For any angle θ, we have

(i) sin $(-\theta)$ = –sin θ

(ii) cosec $(-\theta)$ = –cosec θ

(iii) cos $(-\theta)$ = cos θ

(iv) sec $(-\theta) = \sec\theta$

(v) tan $(-\theta) = -\tan\theta$

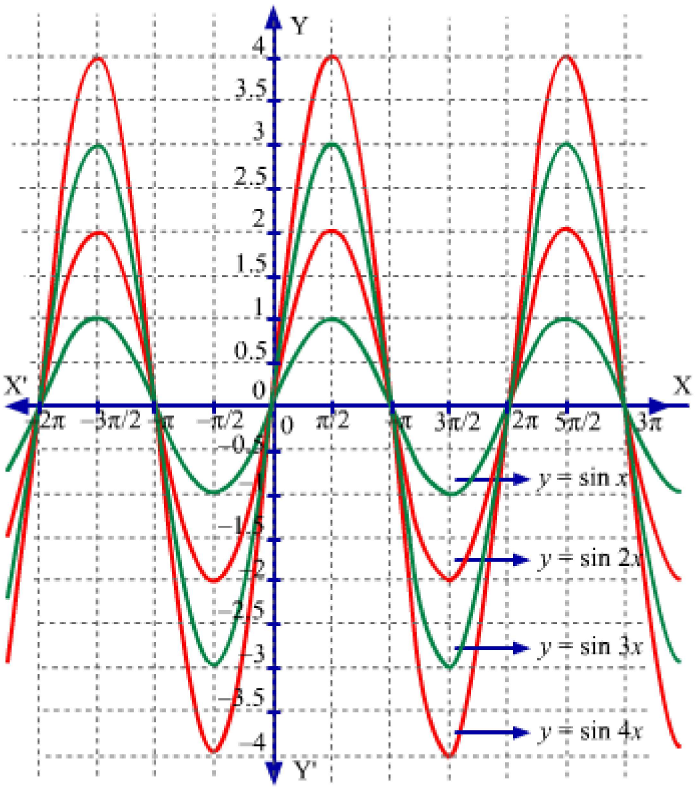
(vi) cot $(-\theta) = -\cot\theta$

- **Domain and Range of trigonometric functions:**

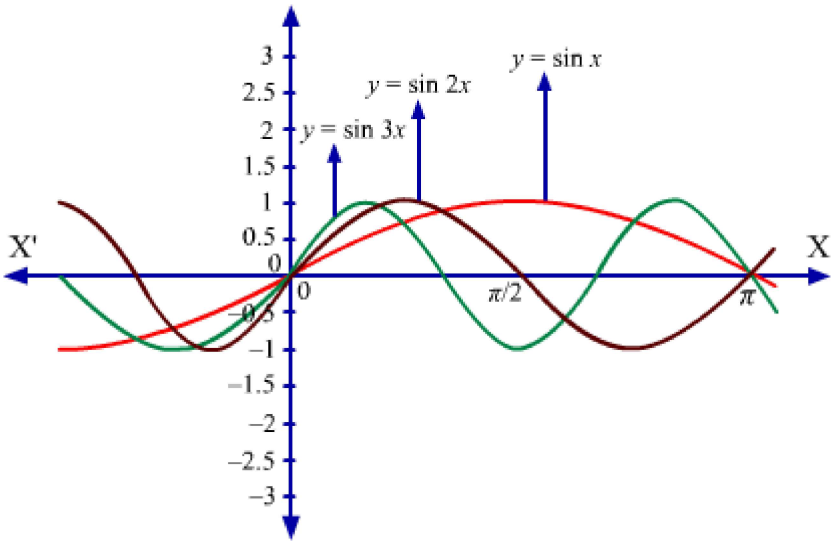
Trigonometric function	Domain	Range
sin x	**R**	[-1, 1]
cos x	**R**	[-1, 1]
tan x	**R** - $\left\{x : x = \frac{(2n+1)\pi}{2}, n \in Z\right\}$	**R**
cot x	**R** - $\{x : x = n\pi, n \in \mathbf{Z}\}$	**R**
sec x	**R** - $\left\{x : x = \frac{(2n+1)\pi}{2}, n \in Z\right\}$	**R** - [-1, 1]
cosec x	**R** - $\{x : x = n\pi, n \in \mathbf{Z}\}$	**R** - [-1, 1]

Graphs of Transformed Trigonometric Functions:

- Graphs of functions of the type y = ***a* sin*x***:

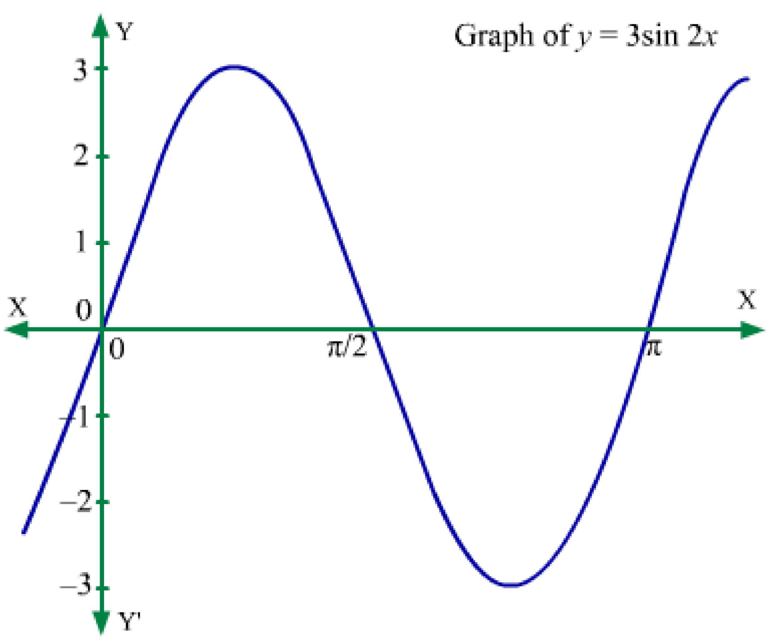


- Graphs of functions of the type ***y* = sin*bx***:

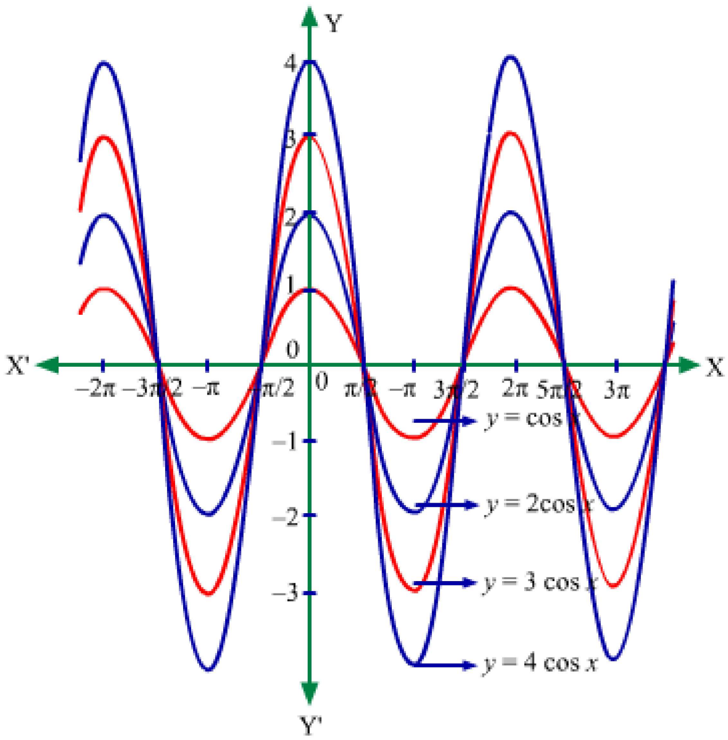


- Graphs of functions of the type $\boldsymbol{y = a \sin bx}$:

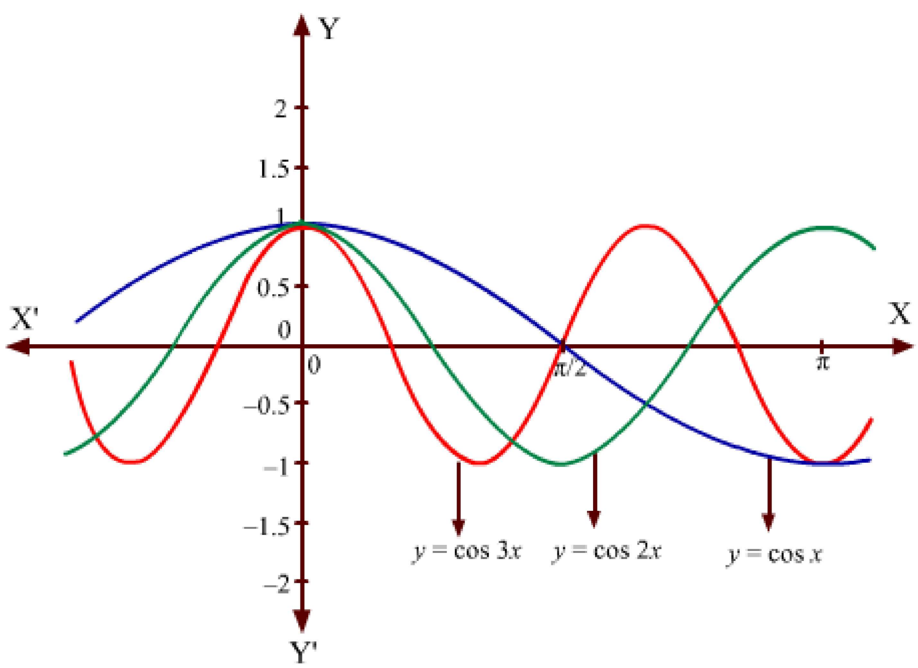
It is easy to understand such a function with the help of an example. Consider the function $y = 3 \sin 2x$.



- Graphs of functions of the type $\boldsymbol{y = a \cos x}$:



- Graphs of functions of the type $y = \cos bx$:



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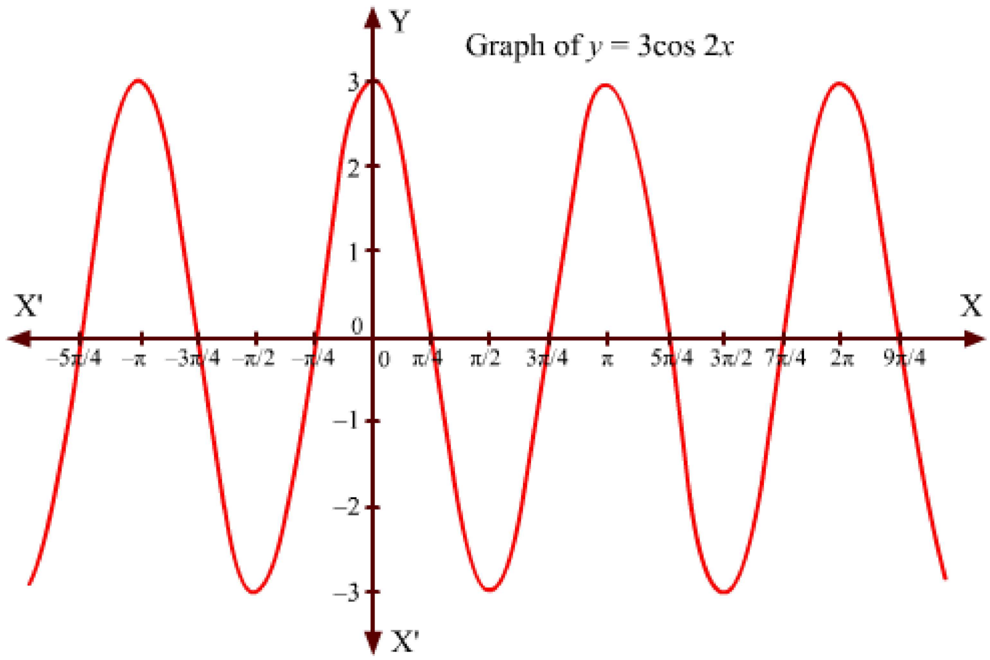
Graph of y = 3cos 2x
Y
X'
X
X'
3
2
1
0
–1
–2
–3
–5π/4
–π
–3π/4
–π/2
–π/4
0
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
2π
9π/4